

## Charmonium Spin Dependent Masses and Leptonic Decay Widths

DHIRENDRA SINGH

Department of Physics  
D.B.S. (P.G.) College, Kanpur, U.P., INDIA

(Received on : April 22, 2012)

### ABSTRACT

The masses of charmonium S, P and D-states, Leptonic decay widths have been computed in an alternative potential model chosen in a combination of Hulthen plus linear potential. The numerical solution of Hamiltonian and the spin hyperfine, spin – orbit, and tensor interaction terms are employed to compute the spectroscopy of  $c\bar{c}$  mesons. The outcome in comparison with the experimental and with the predictions from other theoretical models is presented.

**Keywords:** Charmonia, meson spectroscopy, masses and lepton decay widths.

### INTRODUCTION

There is a wealth of experimental data in hadron spectroscopy that has emerged from a number of experimental facilities such as CLEO, BES, BaBar Belle, CDF, DO, NA60, E815 etc. world over. They have been able to collect huge data samples in the heavy flavour sector, the recent discoveries of conventional and observation of new states<sup>1</sup> have led to renewed interest in quarkonium systems. Since our limited knowledge about exact form of QCD potential, one has to use phenomenological potential models to predict the hadronic properties.

There exists many potential models these models are either relativistic<sup>2-4</sup> or non-relativistic<sup>5-6</sup>. The Hamiltonian of these quark models usually contains three parts. The kinetic energy, the confinement potential, and a hyperfine interaction term. The non-relativistic quark model [NRQM] have proven to be very successful in describing hadronic properties. In the present work, we calculate the S, P and D wave charmonium spectra using a non relativistic potential model. our model consists of a Hulthen and confining linear potential. the model parameters and the wave function that reproduce the mass

spectra are used to study their decay properties.

## THEORETICAL FRAME WORK

For the study of heavy-heavy bound states systems such as  $c\bar{c}$ , we consider a non-relativistic Hamiltonian given by<sup>7</sup>

$$H = M + \frac{P^2}{2m} + V(r) \quad (1)$$

$$\text{Where } M = m_Q + m_{\bar{Q}}, \quad m = \frac{m_Q + m_{\bar{Q}}}{2},$$

$m_Q$  and  $m_{\bar{Q}}$  are the mass parameters of quark-antiquark respectively,  $P$  is the relative momentum of each quark and  $V(r)$  is the quark antiquark potential.

The static  $Q\bar{Q}$  has been chosen here in the form

$$V(r) = V_H(r) + V_C(r) + V_{BF}(r) \quad (2)$$

Where the  $V_H(r)$  Hulthen potential<sup>8</sup>

$$V_H(r) = -\frac{V_o \exp(-r/a)}{[1 - \exp(-r/a)]} \quad (3)$$

The confining part of potential  $V_C$  is given by

$$V_C(r) = cr \quad (4)$$

In order to obtain full spin dependent spectra between different degenerative mesonic

states. Briet-Fermi correction term is added to the potential. Where

$$V_{BF}(r) = V_{LS}(r)(\vec{L} \cdot \vec{S}) + V_\gamma(r) \left[ S(S+1) - \frac{3(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^2} \right] + V_{ss}(r) \left[ S(S+1) - \frac{3}{2} \right] \quad (5)$$

Here  $V_{LS}(r)$  is the spin orbit term,  $V_r(r)$  the tensor term and  $V_{ss}(r)$  represents spin spin term derived by vector and scalar parts of the potential. The coefficient of these spin-dependent terms of eqn.(5) can be written in terms of the vector and scalar parts of the static potential  $V_C(r)$ <sup>9</sup>.

$$V_{LS}(r) = \frac{1}{2m_Q m_{\bar{Q}} r} \left( 3 \frac{dV_r}{dr} - \frac{dV_s}{dr} \right) \quad (6)$$

$$V_\gamma(r) = \frac{1}{6m_Q m_{\bar{Q}} r} \left( 3 \frac{d^2 V_v}{dr^2} - \frac{1}{r} \frac{dV_r}{dr} \right) \quad (7)$$

$$V_{ss}(r) = \frac{1}{3m_Q m_{\bar{Q}} r} \Delta^2 V_v \quad (8)$$

The spin dependent part contains three types of interaction term, such as spin-spin, the spin - orbit and tensor part given by<sup>9,10</sup> using this complete spin dependent potential we have solved the Schrödinger equation with the Hamiltonian equation (1). The model parameters and the wave function that reproduce the mass spectra are used to investigate the leptonic decay widths of  $c\bar{c}$  mesons. The parameters of the model

which we have taken are the masses of  $m_c=1.28$  GeV, the strong coupling constant  $\alpha_s=0.6$ .

Without any additional free parameters, using the Van-Royen-Weisskopf formula for computing leptonic decay widths<sup>11</sup> without radiative correction term, we have computed leptonic decay widths and are tabulated in table 2 along with other theoretical as well as experimental values.

**Table1: Masses (in GeV) of  $c\bar{c}$  in comparison with PDG and other theoretical models**

State	Present	PDG[12]	[13]	[5]	[14]
$1^1S_0$	2.976	2.980	2.976	2.979	3.088
$1^3S_1$	3.099	3.097	3.099	3.096	3.168
$2^3S_0$	3.615	3.637	3.533	3.588	3.669
$1^3P_0$	3.466	3.415	3.466	3.424	3.488
$1^1P_1$	3.514	3.511	3.514	3.510	3.520
$1^3P_1$	3.514	3.526	3.526	3.526	3.536
$1^3D_1$	3.860	-	3.860	3.798	3.789
$1^1D_2$	3.844	-	3.844	3.811	3.803
$1^3D_2$	3.854	-	3.854	3.813	3.804
$1^3D_3$	3.830	-	3.830	3.815	3.809

**Table 2: Leptonic decay widths (in KeV) of  $c\bar{c}$  systems.**

State	Cal.	Expt.[15]	[16]	[17]
$J/\psi(1^3S_1)$	5.429	$5.55\pm0.14$	7.82	$6.72\pm0.49$
$\psi(2^3S_1)$	2.280	$2.48\pm0.06$	3.83	$2.26\pm0.19$
$\psi(3^3S_1)$	0.736	$0.86\pm0.07$	2.79	$1.45\pm0.07$
$\psi(4^3S_1)$	0.291	$0.58\pm0.07$	2.19	$0.52\pm0.02$

## CONCLUSION

In this paper we have computed the spin dependent mass spectra and the leptonic decay widths of various  $c\bar{c}$  states. The computed values of  $c\bar{c}$  systems are shown in table 1 and 2. The results obtained by us clearly show that the potential model consisting of Hulthen and linear potential provides a better agreement with the experimental data on the masses of  $c\bar{c}$  systems. Further our results for the leptonic decay widths also show better agreement with the experimental values and the results predicted by other theoretical models.

## REFERENCES

1. N. Brambilla, NRQCD and Quarkonia, *arXiv; hep-ph/0702105v2* (2007).
2. D. Gromes *Nucl. Phys. B* 130, 18 (1977).
3. O.W. Greenberg *Ann. Rev. Nucl. Part. Sci.* 28, 327 (1978).
4. E. Eichten, K. Gotfried, J. Kinoshita, K.D. Lave and T.M. Yan, *Phys. Rev. D* 17, 3090 (1978).
5. D. Ebert, R. N. Fanstov and V.O. Galkin *Phys. Rev. D* 67, 014027 (2003).
6. O. Lakhina and E. S. Swanson. *Phys. Rev. D* 74 014012 (2006).
7. A. K. Rai and P. C. Vinodkumar. *Pramana J. Phys.* 66 953, (2006).
8. S. Flugge, *Practical Quantum Mechanics* (Narosa Publishing House, New Delhi) 175 (1979).
9. Wolfgang Lucha, Franz F. Schoberl, *arXiv; hep-ph/9601263 V1* (1996).
10. M.B. Voloshin, *Prog. Part. Nucl. Phys.* 61, 455, *arXiv; hep-ph/0711.4556v3* (2008).
11. R. Van Royen and V.F. Weisskopf *Nuovo Cim. A* 50 617 (1967).
12. K. Nakamura *et.al.*, Particle Data Group, *J. Phys. G: Nucl. Part. Phys.* 37, 075021 (2010).
13. B. Patel and P. C. Vinodkumar, *J. Phys. G: Nucl. Part. Phys.* 36, 035003 (2009).
14. O. Lakhina and S. E. Swanson. *Phys. Rev. D* 74 014012 (2006).
15. Y. M. Yao *et. al.* [Particle Data Group]; *J. Phys. G* 33, 1 (2006)
16. Eichten *et. al.*, *Phys. Rev. D* 17, 3090 (1978).
17. S M Ikhdair and R Sever, *Int. J. Mod. Phys. A* 21, 3989 (2006).